Research on the Probability of Anti-Submarine Aerial Depth Charge Hits Based on Particle Swarm Optimization

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Abstract: This paper investigates the probability of anti-submarine aerial depth charge hits based on Particle Swarm Optimization (PSO). The study initially analyzes the horizontal coordinate positioning of depth charges and establishes a maximum kill area model after determining the geometric relationship between the depth charge's lethal radius and the submarine then discusses two detonation methods of depth charges: fixed depth fuse detonation and contact fuse detonation, and analyzes their impact on hit probability. The paper further constructs a depth charge hit probability model, considering the depth positioning error of the submarine's center position, and discusses the impact of different detonation depths on hit probability. Finally, the detonation depth of the depth charge is optimized to maximize the hit probability using the Particle Swarm Optimization algorithm. The results indicate that by optimizing the deployment strategy and detonation depth of depth charges, the efficiency and success rate of anti-submarine operations can be effectively improved.

1. Introduction

In the realm of modern naval warfare, the effectiveness of anti-submarine operations is paramount for maintaining maritime security and dominance. A critical aspect of these operations is the deployment of aerial depth charges, which are designed to engage and neutralize submerged threats. The probability of a successful hit with these depth charges is influenced by a multitude of factors [1], including the accuracy of targeting, the detonation mechanisms, and the environmental conditions. To enhance the efficacy of depth charge deployment, there is a need for a robust method to optimize the probability of a hit [2], which is where Particle Swarm Optimization (PSO), a powerful machine learning algorithm, comes into play.

PSO is renowned for its ability to solve complex optimization problems by simulating the social behavior of a swarm. In the context of anti-submarine warfare, this algorithm can be utilized to determine the optimal detonation depth and positioning of depth charges, thereby maximizing the likelihood of a successful engagement with the target submarine. This paper presents a comprehensive study that leverages PSO to analyze and improve the probability of depth charge hits, offering a significant contribution to the field of naval defense strategy [3, 4].

2. Optimization of Single Depth Charge Hit Probability

2.1. Analysis of Horizontal Coordinate Positioning

After determining the depth range of the deep bomb, we discuss whether the deep bomb should be detonated by fixed depth fuze or contact fuze from the horizontal direction. After comprehensive consideration, we get the maximum hit bomb delivery plan. Among them, when considering the horizontal hit scheme, we first determine the position coordinates of the horizontal drop of the deep bomb, and then build the maximum hit area through the geometric relationship between the kill radius of the deep bomb and the submarine, so as to give the optimal scheme.

Let the position coordinate of the deep projectile drop be (x, y, z), and the positioning error of the

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known submarine center position depth $z_0 = 150m$ is free, while the positioning error of the horizontal coordinate (x, y) is present and follows the normal distribution $N(0, \sigma^2)$, where the standard deviation of the horizontal positioning $\sigma = 120m$, and the probability density function of the standard normal distribution is [5]:

$$\begin{cases} f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{x^2}{2\sigma^2}\right) \\ f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{y^2}{2\sigma^2}\right) \end{cases} \tag{1}$$

Horizontal coordinates are independent random variables, so the expression of the joint probability density function of horizontal coordinates is:

$$f(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \tag{2}$$

Therefore, after the depth range of the deep bomb is determined, the horizontal position coordinate range of the deep bomb is determined, and it is used as the integration area, and the joint density function $f(x, y)$is integrated, and the projectile hit probability can be calculated.

After the anti-submarine aircraft finds the underwater submarine target, it attacks the target by launching deep bombs, and it will fall vertically after deep projectiles into the water. It is known that the killing radius of the deep bomb is r_0=20m. If the deep bomb is assumed to be a particle, its killing range will appear as a sphere, as shown in Figure 1.

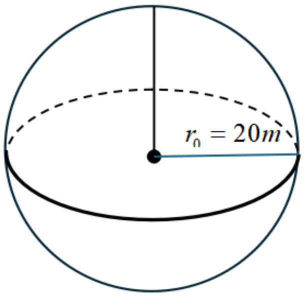

Figure 1 Deep charge maximum lethal range diagram in seawater

The main body of the submarine is simplified into a cuboid with length $l = 100m$, width $w = 20m$ and height $h = 25m$. Its central position coordinate is (x, y, z). The known positioning error $\sigma = 120m$ exists in the two horizontal coordinates (x, y) of the central position of the submarine, which is much larger than the length $l = 100m$ and width $w = 20m$ of the main body of the submarine. Therefore, even if the horizontal positioning coordinates of the deep projectile are set above the submarine, it is still possible to miss the target.

Since the deep bomb uses double fuze to detonate, its detonating rule can be understood as follows: if the deep bomb touches the object in the falling process, the trigger fuze will be activated to detonate, If the object is not touched during the fall, the fixed depth fuze is activated after reaching the specified depth. There are two ways to detonate a deep bomb fuse:

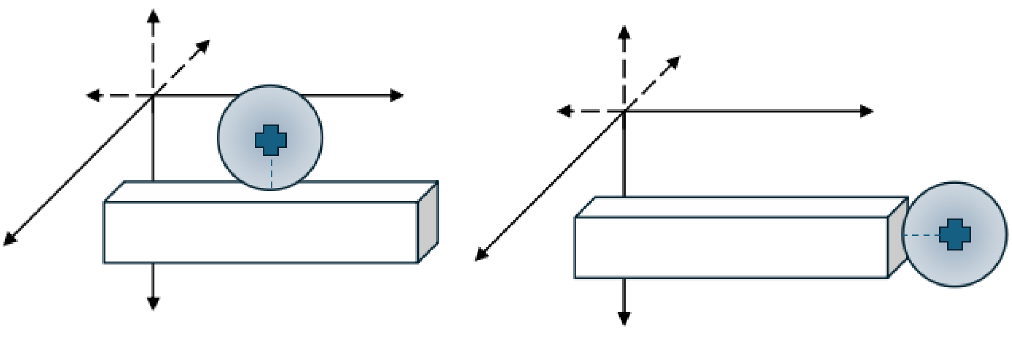

Figure 2 Schematic diagram of depth fuse detonation

(1) Fixed depth fuze detonation: the deep bomb body does not touch the surface of the submarine, but has reached the maximum killing distance r_0, as shown in Figure 2.

(2) Trigger fuse detonation: the deep bomb body has touched the upper surface of the submarine, regardless of whether the specified depth has been reached, the contact triggers the fuze instantly, as shown in Figure 3.

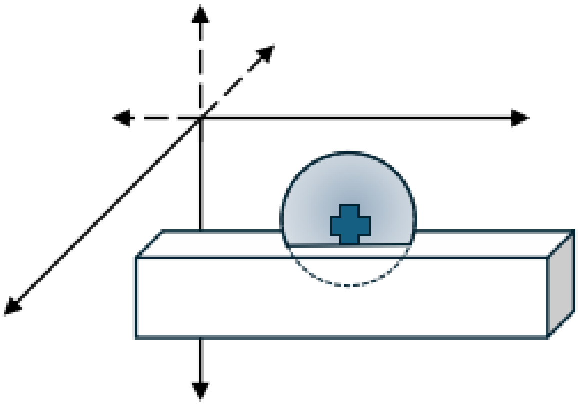

Figure 3 Schematic diagram of impact fuse

According to the above analysis, we draw a conclusion: no matter whether the landing point of the deep bomb falls on the area directly above the submarine, as long as the deep bomb touches the upper surface of the submarine, or within the geometric structure of the maximum damage range of the deep bomb, it can be said that the target is successfully hit. Therefore, the impact area can be divided into two cases of vertical hit the submarine and side hit the submarine.

When the deep bomb hits the area directly above the submarine vertically, the range of the deep bomb's landing point should be consistent with the surface area of the submarine's upper surface. At this time, the projection on the submarine and xoy plane can be regarded as the integral region D_{inside} (see Figure 4), that is, the integral region of the joint density probability function $f(x,y)$ is D_{inside}, and its hit probability is:

$$P_{inside} = \int_{-50}^{50} \int_{-10}^{10} f(x,y)\, dydx \tag{3}$$

When the deep bomb fails to fall in the area directly above the hit submarine and falls from the side, the maximum damage range of the deep bomb can touch the surface of the submarine. At this time, the projection on the plane of the submarine and xoy can be regarded as an integral region $D_{outside}$, and the joint density probability function $f(x,y)$ can be integrated to obtain its hit probability $P_{outside}$:

$$P_{outside} = 4\left(\int_0^{\frac{l}{2}} dx \int_{\frac{w}{2}}^{\frac{w}{2}+r_0} f(x,y)dy + \int_{\frac{l}{2}}^{\frac{l}{2}+r_0} dx \int_0^{\frac{w}{2}} f(x,y)dy + \int_{\frac{l}{2}}^{\frac{l}{2}+r_0} dx \int_{\frac{w}{2}}^{\frac{w}{2}+\sqrt{r^2-\left(x-\frac{l}{2}\right)^2}} f(x,y)dy\right) \tag{4}$$

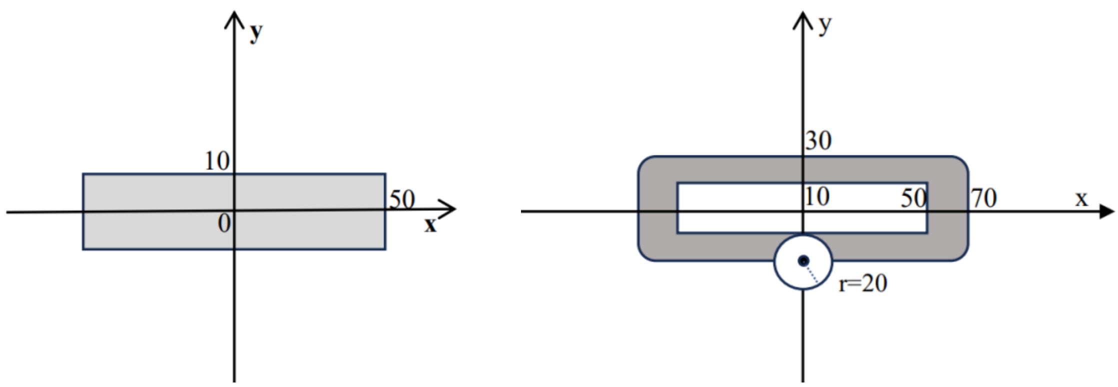

Figure 4 Vertical hit submarine range schematic (left), vertical hit submarine range schematic (right)

From the above analysis, we know that when the horizontal drop position coordinate (x, y) of the deep bomb changes, the hit probability will also change accordingly. To solve this problem, we traverse all the hittable horizontal coverage coordinates and integrate the joint density probability function to calculate the drop probability under each coordinate. The target coordinate system is established with the upper surface of the submarine as the center. Since the integration area is symmetric about the X and Y axes, it is only necessary to conduct probability statistics for the first quadrant.

Where the value range of the x-axis is $[0, \frac{l}{2}]$, the value range of the y-axis is $[0, \frac{w}{2}]$, and the traversal step size is set to 1, the following probability calculation formula can be obtained:

$$\int_0^{\frac{l}{2}+r_0-a} \int_0^{\frac{w}{2}+r_0-b} f(x, y) dy dx \tag{5}$$

The above model is solved by matlab software. When the probability of deep bullet hit is the maximum, the coordinate of the horizontal point should be $(x_0, y_0) = (0,0)$, so the deep bullet drop point is determined to be (x_0, y_0, z).

2.2. Construction of the Depth Charge Hit Probability Model

It is known that the position depth of the submarine center is $z_0 = 150m$, and there is no positioning error. Based on the submarine height $h_0 = 25m$, the submarine upper surface depth positioning value $d_{top} = z_0 - \frac{h_0}{2}$ (m) and lower surface depth positioning value $d_{down} = z_0 + \frac{h_0}{2}$ (m), the detonation depth of the deep bomb is set as z_{burst} (m), so the deep bomb falling vertically within the effective hit range can be divided into four deep intervals due to the difference in its detonation mode as shown in Figure 5.

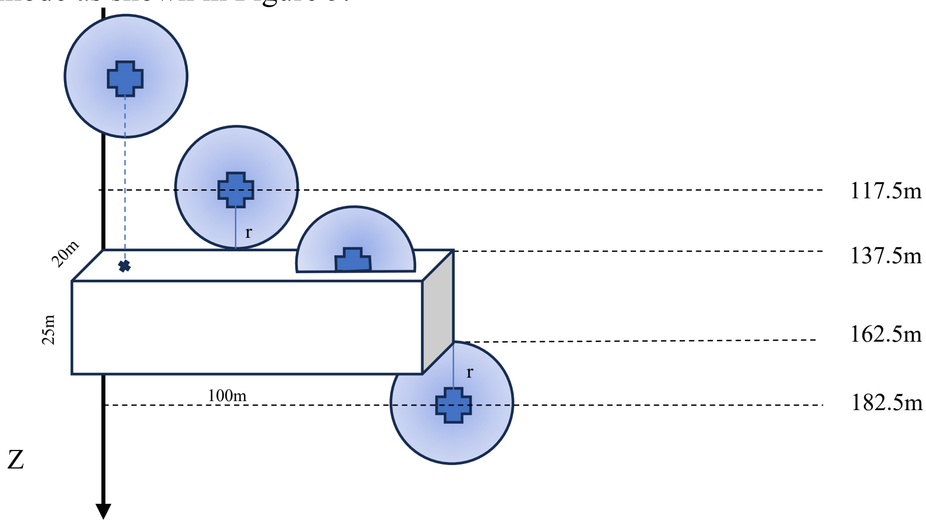

Figure 5 Schematic diagram of different detonation modes of depth charges

When $z_{burst} < d_{top} - r_0$ that is, when the deep bomb reaches the fixed depth value and triggers the fixed depth fuze, the maximum damage range of the deep bomb has exceeded the effective hit range, so the probability of the deep bomb hitting the submarine is 0.

When $d_{top-r_0} \leq z_{burst} < d_{top}$, the deep bomb only has the fixed depth fuze detonation mode, and the deep bomb drop point is located in the projection of the submarine and xoy plane. After reaching the preset depth of explosion, the submarine can be hit.

When $d_{top} \leq z_{burst} \leq d_{down} + r_0$, there are two detonating modes of trigger fuze and fixed depth fuze: when the deep bomb falls on the side of the submarine and is within the effective hitting range, the fixed depth fuze detonates, The deep bomb falls on the upper surface of the submarine, but the preset depth of the explosion has not been reached, and the trigger fuse is activated when it contacts the submarine.

When $d_{down} + r_0 < z_{burst}$, deep bombs only trigger fuze detonation mode: deep bombs fall on the side of the submarine, affected by the depth, the maximum damage range of deep bombs can no longer hit the submarine, and the hit probability is 0. The deep bomb falls on the upper surface of the submarine, and when the deep bomb dives to the set depth value, it will contact the submarine, and then trigger the fuse to detonate.

Therefore, the horizontal hit P_(level_1) of a deep projectile falling vertically in all effective hit ranges can be summarized as:

$$P_{level\_1} = \begin{cases} 0, & z_{burst} < d_{top} - r_0 \\ P_{outside}, & d_{top} - r_0 \le z_{burst} < d_{top} \\ P_{inside} + P_{outside}, & d_{top} \le z_{burst} \le d_{down} + r_0 \\ P_{inside}, & d_{down} + r_0 < z_{burst} \end{cases} \tag{6}$$

2.3. Model Solving

To sum up, so that the hit probability of the deep bomb is the highest, its detonation depth should be set in the range of $d_{top} \le z_{burst} \le d_{down} + r_0$, and when the deep bomb exploates in this depth range, the probability of hitting the submarine is $P_{vertical} = 1$.

According to the above analysis, it can be concluded that the scheme with the maximum projectile hit probability is: The drop level coordinate of the deep bomb is (0,0), the preset depth of the fixed depth fuze of the deep bomb should be within the range of [137.5,182.5], and there is no error in the positioning of the depth of the known center position of the submarine. Therefore, the optimal drop depth of the deep bomb $z_{brust} = 150m$ is selected, and the following data can be obtained by solving the above model through matlab software:

$$P_{\text{overall\_1}} = P_{level\_1} \times P_{vertical} = 0.20989 \tag{7}$$

3. Deep Charge Hit Probability Considering Depth Error

3.1. Depth Error

On the basis of the above model, the depth positioning error is added, that is, there are positioning errors in all directions of the submarine's center position, so the positioning error $\sigma = 120m$ for the horizontal coordinate (x_0, y_0), the positioning error $\sigma_z = 40m$ for the depth coordinate z_0, and the minimum value z_{min} for the actual depth of the submarine's center position.

Given that the vertical coordinate z follows a unilateral truncated normal distribution $N(h_0, {\sigma_z}^2, l)$, its density function is

$$P_{\text{overall\_1}} = P_{level\_1} \times P_{vertical} = 0.20989 \tag{8}$$

The depth of the submarine's center position is h_0, the minimum actual depth is l, and the density function of the standard normal distribution is ϕ and the standard normal distribution function is Φ.

Since the horizontal probability does not change and the size of the submarine does not change, the impact of the depth z of the deep projectile on the total probability is only rediscussed. The total hit probability is calculated by multiplying the hit probability of the horizontal direction obtained in Chapter 2 by the hit probability of the vertical direction to be obtained:

$$P_{overall\_1} = P_{level} \times P_{vertical} \tag{9}$$

3.2. Model Establishment

Given the actual depth z of the central position of the submarine, since the detonation depth of the deep bomb is z_{burst}, the influence of the detonation depth on the hit probability in question 1 is analyzed, and the P_{level_2} in this question can be obtained: When $z_{burst} < z - \frac{l}{2} - r_0$, the maximum damage range of the deep bomb has exceeded the range of the submarine, and the probability of the deep bomb hitting the submarine is 0. When $z - \frac{l}{2} - r_0 \le z_{burst} < z - \frac{l}{2}$, the deep bomb only has a fixed depth fuze detonation mode. When $z - \frac{l}{2} \le z_{burst} \le z + \frac{l}{2} + r_0$, there are two detonation modes: deep trigger fuze and fixed depth fuze. When $z + \frac{l}{2} + r_0 < z_{burst}$, deep rounds only have trigger fuze firing mode. Therefore, the horizontal hit P_{level_1} of a deep projectile falling vertically in all effective hit ranges can be summarized as

$$P_{level\_1} = \begin{cases} 0, & z_{burst} < d_{top} - r_0 \\ P_{outside}, & d_{top} - r_0 \leq z_{burst} < d_{top} \\ P_{inside} + P_{outside}, & d_{top} \leq z_{burst} \leq d_{down} + r_0 \\ P_{inside}, & d_{down} + r_0 < z_{burst} \end{cases} \tag{10}$$

Therefore, in order to maximize the hit probability of the deep bomb, its detonation depth should be set within the range of $z - \frac{l}{2} \leq z_{burst} \leq z + \frac{l}{2} + r_0$.

After analyzing the above longitudinal depth, the value interval of fuze with maximum probability is selected, and the drop location coordinates of the deep bomb are set as (x_0, y_0, z), where z follows a unilateral truncated normal distribution, and there is standard deviation σ_z in the positioning of the central position depth z, and the minimum value of the submarine deep central position is z_{min}. Therefore, according to the above discovery hit probability:

$$P_{overall\_2} = P_{level} \times P_{vertical} \tag{11}$$

$P_{overall}$ is the total hit probability, P_{level} is the hit probability of horizontal drop, and $P_{vertical}$ is the hit probability of vertical drop. P_{level} can be calculated as $P_{level} = P_{in} + P_{out}$ according to the probability model of horizontal drop in Chapter 2. The expression of the hit probability $P_{vertical}$ for vertical delivery is:

$$P_{\text{vertical}} = \int_{z-\frac{w}{2}}^{z+\frac{w}{2}} \frac{1}{\sigma_z} \times \frac{\phi(\frac{v-h_0}{\sigma_z})}{1-\varphi(\frac{l-h_0}{\sigma_z})} dv \qquad (l < v < +\infty), \tag{12}$$

$\phi(z) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{z^2}{2\sigma_z{}^2}\right)$ as the standard normal distribution density function, $\varphi(z) = \int_{-\infty}^{z} \phi(t)dt$ as a distribution function of standard normal distribution, because z is an unknown quantity, so the optimal solution for z as follows:

$$\max_{z \geq 120} \; P_{overall\_2} \tag{13}$$

It can be seen that this is a single-objective nonlinear optimization problem, so particle swarm optimization can be used to optimize it [6]. Firstly, the position of each particle is evaluated by using the objective function to obtain the fitness value, so as to ensure that the particle searches in the direction of the optimal solution, and then the space range of particle search is restricted to constrain the optimal objective function value. The steps are repeated until the maximum value is found.

3.3. Model Solving Based on Particle Swarm Optimization

According to the above particle swarm optimization problem solved by matlab, in order to maximize the probability of deep bomb hitting the submarine, the fixed depth detonation depth should be $z_{burst} = 150m$, then the probability of deep bomb hitting the submarine is $P_{vertical} = 0.36374$ when the deep bomb bursts at this fixed depth.

If the probability $P_{level\_1} = 0.20989$ is obtained, then the total probability $P_{\text{overall\_2}}$ is:

$$P_{\text{overall\_2}} = P_{level\_1} \times P_{vertical} = 0.076345 \tag{14}$$

4. Conclusion

This study has demonstrated the potential of Particle Swarm Optimization (PSO) in enhancing the probability of anti-submarine depth charge hits. The model developed offers a significant improvement over traditional methods, but there is room for further refinement. Future work will explore the incorporation of real-time data and advanced algorithms to adapt to dynamic ocean conditions and improve the model's predictive power.

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